Solve system of simultaneous equations with 150 variables

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In the following systems of equations, solve only for **real** solutions.

1. Solve :
$$\begin{cases} x_1 + x_2 + \cdots + x_k + \cdots + x_{150} = 0 \\ x_1 - x_2 = x_2 - x_3 = \cdots = x_{k-1} - x_k = \cdots = x_{149} - x_{150} = 1 \end{cases}$$

2. Solve :
$$\begin{cases} x_1 + 2x_2 + 3x_3 + \cdots + 150x_{150} = a_1 \\ x_2 + 2x_3 + 3x_4 + \cdots + 150x_1 = a_2 \\ \cdots\cdots\cdots \\ x_{150} + 2x_1 + 3x_2 + \cdots + 150x_{149} = a_{150} \end{cases}$$, where a_k are constants.

3. Solve :
$$\begin{cases} x_1 + x_2 + \cdots + x_{150} = 1 \\ x_1 + x_3 + \cdots + x_{150} = 2 \\ x_1 + x_2 + x_4 + \cdots + x_{150} = 3 \\ \cdots\cdots\cdots \\ x_1 + x_2 + \cdots + x_{149} = 150 \end{cases}$$

4. Solve :
$$\begin{cases} x_1 - x_2 - x_3 \ldots - x_{150} = 2k \\ -x_1 + 3x_2 - x_3 - \cdots - x_{150} = 4k \\ -x_1 - x_2 + 7x_3 - \cdots - x_{150} = 8k \\ \cdots\cdots\cdots \\ -x_1 - x_2 - x_3 - \cdots + (2^{150} - 1)x_{150} = (2^{150})k \end{cases}$$,

where k is a non-zero constant.

5. Solve :
$$\begin{cases} x_1 + x_2 = a_1 \\ x_2 + x_3 = a_2 \\ x_3 + x_4 = a_3 \\ \cdots\cdots\cdots \\ x_{149} + x_{150} = a_{149} \\ x_{150} + x_1 = a_{150} \end{cases}$$, where a_k $(k = 1, 2, \ldots, 150)$ are constants.

6. Solve :
$$\begin{cases} x_1x_2 \ldots x_{150} = 1 \\ x_1 - x_2x_3 \ldots x_{150} = 1 \\ x_1x_2 - x_3x_4 \ldots x_{150} = 1 \\ \cdots\cdots\cdots \\ x_1x_2 \ldots x_{149} - x_{150} = 1 \end{cases}$$

7. Solve :
$$\begin{cases} x_0x_1x_2 = x_0 + x_1 + x_2 \\ x_1x_2x_3 = x_1 + x_2 + x_3 \\ \cdots\cdots\cdots \\ x_{149}x_{150}x_0 = x_{149} + x_{150} + x_0 \\ x_{150}x_0x_1 = x_{150} + x_0 + x_1 \end{cases}$$